

Errata of the book *Intermediate Spectral Theory and Quantum Dynamics, Progress in Mathematical Physics 54, Birkhäuser, Basel, 2008, by César R. de Oliveira.*

Corrections ordered by page numbers

- page 15, line -10. The left hand side of the polarization identity should read:  $\langle \eta, \xi \rangle$
- page 143. The top of page should read:
  2. For bounded operators  $A, B$  and all  $n \in \mathbb{N}$ , one has (expand the r.h.s.)

$$A^n - B^n = \sum_{j=0}^{n-1} A^j (A - B) B^{n-1-j},$$

and the choices  $A = e^{-iT/n} e^{-iS/n}$ ,  $B = e^{-it(T+S)/n}$  imply that, for any  $n \in \mathbb{N}$ ,

$$(e^{-iT/n} e^{-iS/n})^n - (e^{-it(T+S)/n})^n = \dots$$

- page 150, line +1. The form  $b_1$  should be supposed to be closed in the KLMN Theorem.
- page 173. The proof of Theorem 7.1.13 needs to be corrected, since it was implicitly assumed that  $\text{rng } \rho_1|_{\text{dom } T_{\hat{U}}}$  is also dense in  $\mathbf{h}$ . In fact, the very definition of boundary triples (Definition 7.1.11) should include the assumption that the map

$$\begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} : \text{dom } T^* \rightarrow \mathbf{h} \times \mathbf{h}$$

is onto (instead of the ranges of both  $\rho_1$  and  $\rho_2$  be dense in  $\mathbf{h}$ ). The proof of Theorem 7.1.13 then goes as follows.

Given a unitary  $\hat{U}$ , as in the original proof  $T_{\hat{U}}$  is hermitian and it will be argued that  $T_{\hat{U}}^* \subset T_{\hat{U}}$ , so that  $T_{\hat{U}}$  is self-adjoint. Let  $\eta \in \text{dom } T_{\hat{U}}^*$ .

The relation  $0 = \langle \rho_1(\eta), \rho_1(\xi) \rangle - \langle \rho_2(\eta), \rho_2(\xi) \rangle$ , for all  $\xi \in \text{dom } T_{\hat{U}}$  (see page 174), implies the orthogonality

$$0 = \left\langle \begin{pmatrix} \rho_1(\xi) \\ \rho_2(\xi) \end{pmatrix}, \begin{pmatrix} \rho_1(\eta) \\ -\rho_2(\eta) \end{pmatrix} \right\rangle,$$

and since the graph  $\mathcal{G}(\hat{U})$  is a (closed) subspace of  $\mathbf{h} \times \mathbf{h}$ , the set

$$\left\{ \begin{pmatrix} \rho_1(\xi) \\ \rho_2(\xi) \end{pmatrix} : \xi \in \text{dom } T_{\hat{U}} \right\} = \left\{ \begin{pmatrix} \rho_1(\xi) \\ \rho_2(\xi) \end{pmatrix} : \xi \in \text{dom } T^*, \rho_2(\xi) = \hat{U}\rho_1(\xi) \right\}$$

coincides with the graph of  $\hat{U}$ . By Lemma 2.1.15 and the above orthogonality,  $\begin{pmatrix} \rho_2(\eta) \\ \rho_1(\eta) \end{pmatrix} \in \mathcal{G}(\hat{U}^*)$ , that is,  $\hat{U}^*\rho_2(\eta) = \rho_1(\eta)$ , and so  $\rho_2(\eta) = \hat{U}\rho_1(\eta)$ . Therefore,  $\eta \in \text{dom } T_{\hat{U}}$ .

This modification of the definition of boundary triple also affects some arguments in page 198, since the ranges of both  $\rho_1(\psi)$  and  $\rho_2(\psi)$  are not the whole space  $\mathbf{h} = L^2(S)$ . So the conclusion that  $H_U$  is self-adjoint holds true under the following additional assumptions: (i)  $-1 \in \rho(U)$  and (ii) if  $\psi \in \mathcal{H}^1(S)$ , then  $(\mathbf{1} + U)^{-1}(\mathbf{1} - U)\psi \in \mathcal{H}^{1/2}(S)$  (see Theorem 4.5 of J. Behrndt, M. Langer, V. Lotoreichik, Spectral estimates for resolvent differences of self-adjoint elliptic operators. Preprint: arXiv:1012.4596).

Remark: In the particular case  $U = e^{iu(\varphi)}$  (see item 4 in page 199), to guarantee that  $H_U$  is self-adjoint it is sufficient to impose that (j)  $-1 \notin \text{essrng } e^{iu(\varphi)}$  and (jj)  $\nabla e^{iu(\varphi)} \in L^\infty(S)$ , which ensure (i) and (ii) above, respectively.

(I warmly thank Jussi Behrndt and Vladimir Lotoreichik, from Austria, for sharing their expertise with me and this important correction.)

- page 227, line +10. The central part of the equation should read

$$\mathbf{1} - \frac{1}{2} (W_t(x) + W_t(x)^2)$$

- page 248, lines +3 and +4. It should be  $S = S_1 + iS_2$  with  $S_2 = \frac{i}{2}(S^* - S)$ .

- page 264, line +10. The set  $\Lambda_\varepsilon$  should read  $\Lambda_\varepsilon = (a + \varepsilon, b - \varepsilon)$ .
- page 265, line +17. It should be  $\|f_\varepsilon(T_n)\| < 1$  instead of  $\|f_\varepsilon\|_\infty < 1$ .
- page 348, last equation in the proof of Proposition 12.6.1. It should be

$$Y = \bigcap_{j,n \in \mathbb{N}} \bigcup_{t \in \mathbb{N}} \left\{ T \in X : \langle p_{\psi_j}^T \rangle(t) < \frac{1}{n} \right\}.$$

(I thank Jonás Arista, from Mexico, for this correction)